Collimation of High-Speed Flow with Thermal Spread Losses

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I. Introduction

A NEUTRAL beam collimator has been designed for spaceflight applications on a mass spectrometer inlet system. The collimator will allow selective views of a 10 deg half-angle conical volume of space, so that backscattered molecules from within this volume can be measured and related to the column densities of outgassing species about the spacecraft. The collimator to be discussed in this paper is specifically for use in the Space Shuttle Induced Environment Contamination Monitor (IECM) system, which will be on board the first seven orbital flight tests of the shuttle. Only the collimator will be discussed in this paper.

The collimator consists of a canister about 12.7 mm in diameter and 7.6 mm long, which houses a bulk getter material for removing gas. Gas enters a 3 mm diam orifice on the face of the canister at the center or axis of the cylinder. The exit of the collimator is a 3 mm orifice at a distance of 17 mm from the inlet orifice. The collimator output vs input angle has a response curve which is almost triangular and is the result of convoluting two circular orifices and determining the common area of each vs angle of view from the normal to the orifices.

Between the entrance and exit orifices are placed two chevron baffles which aid in directing the unwanted stream particles into the getter pump volume. Molecules which do not have a view from entrance to exit orifices are ideally directed into the pump section and removed from consideration.

II. Theory

The main purpose of the collimator is to limit the view of space to 0.1 sr or to a 10 deg half-angle. The collimator does this with a response such that for a random flux flowing to the collimator, approximately half of the flux within 0.1 sr flows through the collimator. In the case of a directed flux, such as the flux that occurs when the collimator is pointed in the direction of motion of the space vehicle, one must consider the loss in the collimator due to the thermal spread of the molecules that are streaming through. Since the secondary purpose is to use the data received during the direct streaming measurements to determine the ambient conditions, the loss to the collimator must be known for each species.

In general, for each species of interest, the ambient conditions can be determined from a continuity equation of the form

$$N_0 = N_i / 2\sqrt{\pi\beta}\, U_0 \cos\alpha\, L(S,\alpha) \tag{1}$$

where

N_0 = ambient atom or molecule density
N_i = ion source number density
$\left(\frac{1}{\beta}\right)^{1/2}$ = ion source thermal speed of atom or molecule $= \sqrt{2kT/m}$
k = Boltzmann's constant
T = temperature
m = mass of atom or molecule
U_0 = vehicle velocity
α = angle of attack
L = the loss coefficient to be determined here
S $= U_0(\sqrt{2kT_a/m})^{-1} = U_0\beta^{1/2}$

The geometry for this analysis is shown in Fig. 1. For a flux of known quantity through A_1, we wish to know the flux through the orifice of area A_2 at the back surface. It is assumed that those molecules not directed toward the back surface orifice are removed from consideration by the pumps of the collimator.

Given a gas with a flow velocity of U_0, we assume a Maxwell-Boltzmann distribution of velocities given by [1,2]

$$\frac{1}{N_0}\frac{dN_0}{dV} = f(V\cos\theta) + \left(\frac{\beta}{\pi}\right)^{3/2} V^2$$
$$\times \exp[-\beta(V^2 - 2U_0 V\cos\theta + U_0^2)]\, d\Omega$$

where: θ is from Fig. 1; $d\Omega$ = solid angle = $\sin\theta d\theta d\phi$ for cylindrical symmetry; and subscripts 1 and 2 refer to the front or entrance surface and back or exit surface, respectively; and

$$(V^2 - 2U_0 V\cos\theta + U_0^2) = |\bar{U}_0 - \bar{V}|^2$$

Now

$$\Gamma_{\text{in}} = N_0 U_0 A_1 = \text{parts/s}$$

and

$$\Gamma_{\text{out}} = A_2 N_0 \int_0^{2\pi}\int_0^{\theta_0}\int_0^{\infty} f(V\cos\theta)\, V\cos\theta\sin\theta\, d\theta\, d\phi\, dV$$

or

$$\Gamma_{\text{out}} = A_2 N_0 2\pi\left(\frac{\beta}{\pi}\right)^{3/2}\int_0^{\infty} V^3$$
$$\times \exp - \beta(V^2 + U_0^2)\, dV \left\{\int_{\cos\theta_0}^{1.0} x\exp(2U_0\beta V x)\, dx\right\}$$

This expression is integrable, and the general result is

$$\Gamma_{\text{out}} = \frac{A_2 N_0}{2\sqrt{\pi}}\beta^{-1/2}\{\exp(-S^2)(1-\cos^2\theta_0) + \sqrt{\pi}S[1+\text{erf}S]$$
$$-\sqrt{\pi}\cos^3\theta_0 \exp[-S^2(\sin^2\theta_0)][1+\text{erf}S]\}$$

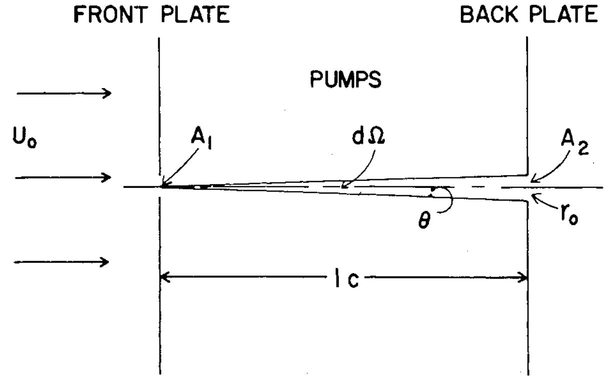

Fig. 1 Collimator flow schematic.

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Index categories: Computational Methods; Rarefied Flows.

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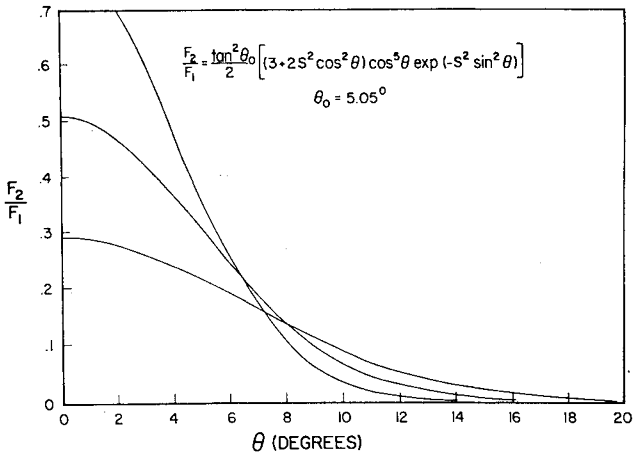

Fig. 2 Flux per unit area, in to out ratio, vs angular dispersion.

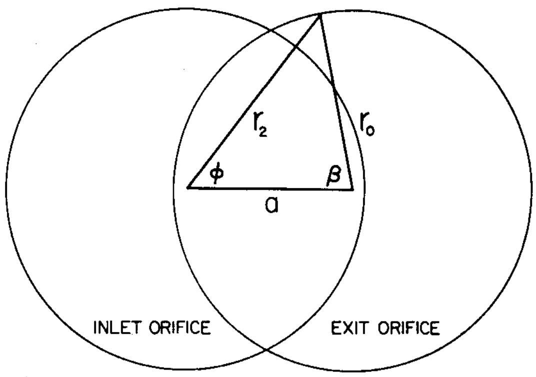

Fig. 3 Geometry for common area intersection considerations.

For

$$S>4, \quad 1+\text{erf}S=2, \quad \exp(-S^2)\simeq 0$$

Therefore

$$\frac{\Gamma_{\text{out}}}{\Gamma_{\text{in}}} = [1-\cos^3\theta_0 \exp(-S^2\sin^2\theta_0)]\frac{A_2}{A_1} \tag{2}$$

where $\tan\theta_0 = r_0/\ell_c$ (see Fig. 1).

Another quantity of interest is the flux distribution at the back plate, or F_2 in part/cm^2 s vs θ.

This is obtained by differentiating the flux ratio with respect to area at the back plate, or

$$\frac{N_2U_2}{N_0U_0} = \frac{F_2}{F_1} = A_1\frac{\partial[1-\cos^3\theta\exp(-S^2\sin^2\theta)]}{\partial A_2}$$

This yields

$$\frac{F_2}{F_1} = \frac{\tan^2\theta_0}{2}[(3+2S^2\cos^2\theta)\cos^5\theta\exp(-S^2\sin^2\theta)] \tag{3}$$

which is plotted in Fig. 2 for several S values expected for the IECM flight, for $A_1 = A_2$ and $\tan\theta_0 = .0882$ or $\theta_0 \simeq 5.04$ deg.

Now with this result the inlet to outlet flux values can be computed for any angle of attack. This is done by using the geometry of Fig. 3. From this figure: α = angle of attack $= \tan^{-1}a/\ell_c; r_2 = (r_0{}^2 + a^2 - 2r_0 a\cos\beta)^{1/2}; \quad \cos\phi = (r_2{}^2 + a^2 - r_0{}^2)/2r_2a$; and $\tan\theta = r/\ell_c$.

In the figure, the flux distribution is symmetric about the center of the inlet orifice. However, we must integrate this

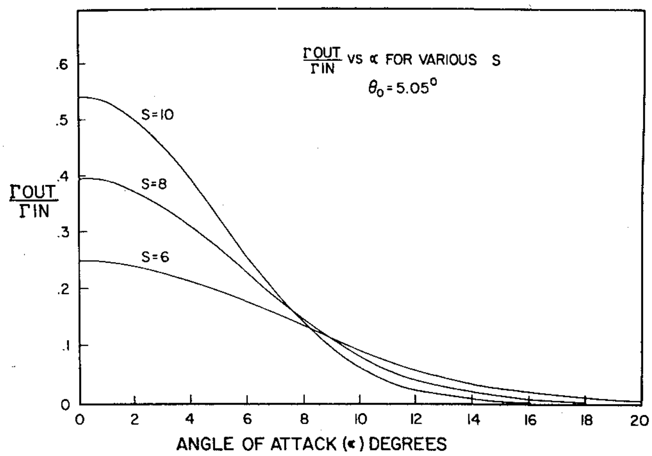

Fig. 4 Flux out to flux in ratio vs angle of attack.

over the outlet orifice area:

$$\frac{\Gamma_{\text{out}}(\alpha)}{\Gamma_{\text{in}}} = \frac{1}{A_1}\int_{\phi(r_{\min})}^{\phi(r_{\max})}\int_{r_{\min}}^{r_{\max}} \frac{F_2}{F_1} r dr d\phi$$

now

$$dr = \ell_c \sec^2\theta d\theta$$

and

$$\frac{d\phi}{d\beta} = \left[\frac{ar_0\cos\beta - r_0^2}{(a^2+r_0^2-2ar_0\cos\beta)}\right]$$

This becomes for $a \leq r_0$

$$\frac{\Gamma_{\text{out}}(\alpha)}{\Gamma_{\text{in}}} = \frac{1}{\pi}\int_0^\pi\left[\cos^3\tan^{-1}\frac{r_2}{\ell_c}\exp\left(-S^2\sin^2\tan^{-1}\frac{r_2}{\ell}\right) - 1\right]\left(\frac{ar_0\cos\beta - r_0^2}{r_2^2}\right)d\beta \tag{4a}$$

and for $a > r_0$

$$\frac{\Gamma_{\text{out}}(\alpha)}{\Gamma_{\text{in}}} = \frac{1}{\pi}\int_0^\pi\left[\cos^3\tan^{-1}\frac{r_2}{\ell_c}\exp\left(-S^2\sin^2\tan^{-1}\frac{r_2}{\ell_c}\right) - \cos^3\tan^{-1}\left(\frac{a-r_0}{\ell_c}\right)\exp\left[-S^2\sin^2\tan^{-1}\left(\frac{a-r_0}{\ell_c}\right)\right]\right] \times \left(\frac{ar_0\cos\beta - r_0^2}{r_1^2}\right)d\beta \tag{4b}$$

These equations were integrated numerically and the results are given in Fig. 4 for $A_1 = A_2$, $r_0 = 1.5$ mm and $\ell_c = 17$ mm. The $\Gamma_{\text{out}}/\Gamma_{\text{in}} = L(S,\alpha)$ is used in Eq. 1 to determine N_0 from the measure N_i.

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[2] Jeans, Sir J., *The Dynamical Theory of Gases,* 4th Ed., Dover Publications, New York, N.Y. 1954, Chap. 2.